8.6 Probability

The Probability of an Event • Mutually Exclusive Events • Independent Events • The Complement of an Event

The Probability of an Event

In Section P.7, you were introduced to the concept of the probability of a simple event. Recall that any happening whose result is uncertain is called an experiment. The possible results of the experiment are outcomes, the set of all possible outcomes of an experiment is the sample space of the experiment, and any subcollection of a sample space is an event.

For instance, when a six-sided die is tossed, the sample space can be represented by the numbers from 1 through 6. For this experiment, each of the outcomes is equally likely.

To describe sample spaces in such a way that each outcome is equally likely, you must sometimes distinguish between various outcomes in ways that appear artificial. Example 1 illustrates such a situation.

EXAMPLE 1 Finding the Sample Space

Find the sample spaces for the following.

a. One coin is tossed.  b. Two coins are tossed.  c. Three coins are tossed.

Solution

a. Because the coin will land either heads up (denoted by \(H\)) or tails up (denoted by \(T\)), the sample space is \(S = \{H, T\}\). Outcome = Heads or Tails

b. Because either coin can land heads up or tails up, the possible outcomes are as follows.

\[HH = \text{heads up on both coins}\]
\[HT = \text{heads up on first coin and tails up on second coin}\]
\[TH = \text{tails up on first coin and heads up on second coin}\]
\[TT = \text{tails up on both coins}\]

Thus, the sample space is \(S = \{HH, HT, TH, TT\}\). Note that this list distinguishes between the two cases \(HT\) and \(TH\), even though these two outcomes appear to be similar. Sequence matters.

c. Following the notation of part (b), the sample space is

\[S = \{HHH, HHT, HTH, HTT, TTH, THT, TTT\}\]
To calculate the probability of an event, count the number of outcomes in the event and in the sample space. The number of outcomes in event \( E \) is denoted by \( n(E) \) and the number of outcomes in the sample space \( S \) is denoted by \( n(S) \). The probability that event \( E \) will occur is given by \( \frac{n(E)}{n(S)} \).

**The Probability of an Event**

If an event \( E \) has \( n(E) \) equally likely outcomes and its sample space \( S \) has \( n(S) \) equally likely outcomes, the probability of event \( E \) is

\[
P(E) = \frac{n(E)}{n(S)},
\]

Because the number of outcomes in an event must be less than or equal to the number of outcomes in the sample space, the probability of an event must be a number between 0 and 1. That is,

\[
0 \leq P(E) \leq 1.
\]

If \( P(E) = 0 \), event \( E \) cannot occur, and \( E \) is called an impossible event. If \( P(E) = 1 \), event \( E \) must occur, and \( E \) is called a certain event.

**EXAMPLE 2** Finding the Probability of an Event

a. Two coins are tossed. What is the probability that both land heads up?

b. A card is drawn from a standard deck of playing cards. What is the probability that it is an ace?

**Solution**

a. Following the procedure in Example 1(b), let

\( E = \{HH\} \)

and

\( S = \{HH, HT, TH, TT\} \).

The probability of getting two heads is

\[
P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}.
\]

b. Because there are 52 cards in a standard deck of playing cards and there are four aces (one in each suit), the probability of drawing an ace is

\[
P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}.
\]